UDC 536.12

EFFECT OF AXIAL HEAT FLUX ON THE TEMPERATURE FIELD OF A HOLLOW CYLINDER

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 5, pp. 597-601, 1965

Solutions are given for two-dimensional problems of heat conduction in a hollow cylinder with quasi-steady heating (constant heating rate) for a boundary condition of the second kind at the inner surface of the cylinder, and boundary conditions of the third or first kind at the outer surface. The effect of axial heat flux on the temperature field of the hollow cylinder is estimated.

Most existing methods for determining thermophysical properties of materials are based on the solution of heat conduction problems that stipulate one-dimensional heat flow in the material examined. The resulting mathematical expression for the temperature field in the specimen is in each case some approximation to the actual field, since it must be assumed that the actually finite dimensions of the sample are infinitely large. On the other hand, this is fully justified by the fact that in one-dimensional problems the expressions for temperature fields are relatively simple in form and usually yield uncomplicated formulas for calculating thermophysical properties. It should also be noted that great difficulty is often encountered in solving two-dimensional problems, and even more so in the three-dimensional case.

It is evident that the experimental realization of methods based on the solution of one-dimensional problems also requires conditions ensuring a one-dimensional temperature field in the specimens with a sufficient degree of accuracy. Absence of these conditions leads to a considerable error, which may escape the notice of the experimenter if the experimental technique does not allow close control of the process, especially since deviation from the boundary conditions of the problem is often disregarded in estimating the error of the method, the values given in the literature often being the instrumental error only.

In the particular case of heating of a cylindrical specimen in a furnace by a radial heat flux, the question inevitably arises of distortion of the one-dimensional temperature field due to heat flux parallel to the cylinder axis. In this case, and indeed for a body of any shape, there are two alternatives: 1) to base the method on the solution of a two-dimensional problem or to introduce empirical correction factors into the calculations, and 2) to select a specific ratio between the dimensions of the test specimen (for a cylinder, the ratio of length to diameter) or to use a shielding device to minimize the influence of axial fluxes on the temperature field at the points of temperature measurement.

The majority of investigators choose the second alternative [1-4]. However, a fairly rigorous theoretical basis is then needed, in each separate case, for the choice of specimen dimensions, since too small a ratio of length to diameter leads to incorrect results, while too large a ratio entails difficulties in preparing specimens of the necessary size, an increase in the size of the experimental apparatus and the demands upon it, and difficulties associated with the installation of temperature sensors, and so on.

The present paper aims at providing a theoretical basis for the choice of optimal dimensions of test specimens in investigations of the thermophysical properties of nonmetallic materials by the method proposed in [5], which is based on the solution of the problem of constant-rate heating of an infinite hollow cylinder with a boundary condition of the third kind at its outer surface and one of the second kind at its inner surface.

To derive the appropriate relations, one must analyze the solution of the following two-dimensional problem:

$$\frac{\partial T}{\partial \mathrm{Fo}} = k^2 \left(\frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right) + l^2 \frac{\partial^2 T}{\partial Z^2}, \tag{1}$$

$$T|_{\mathrm{Fo}=0} = 1, \tag{2}$$

$$\left. \frac{\partial T}{\partial R} \right|_{R=1} = -\frac{\mathrm{Ki}}{k}, \tag{3}$$

$$\left. \frac{\partial T}{\partial R} \right|_{R=k} = \frac{\mathrm{Bi}}{k} \left(1 + \mathrm{Pd\,Fo} - T|_{R=k} \right), \tag{4}$$

$$\left. \frac{\partial T}{\partial Z} \right|_{Z=0} = 0, \tag{5}$$

$$\left. \frac{\partial T}{\partial Z} \right|_{Z=l} = \frac{\mathrm{Bi}}{k} \left(1 + \mathrm{Pd\,Fo} - T|_{Z=1} \right). \tag{6}$$

The solution for quasi-steady heat flow is:

$$T(R,\ Z,\ \mathrm{Fo}) = 1 + \mathrm{Pd\,Fo} - \Phi(R,\ Z), \tag{7}$$

where

$$\Phi(R,\ Z) = 2\sum_{n=1}^{\infty} \varphi_n(R)\,\psi_n(Z), \tag{8}$$

$$\varphi_n(R) = \left[\frac{\mathrm{Pd}\,W_1(\delta_n k)}{\delta_n k} + \frac{2\mathrm{Ki}}{\delta_n \pi}\right] \frac{W_0(\delta_n R)}{\delta_n^2 k^2 W_1^2(\delta_n k)(1 + k^2\delta_n^2/\mathrm{Bi}^2) - W_0^2(\delta_n)]}\ , \tag{9}$$

$$\psi_n(Z) = \frac{\delta_n k\,\mathrm{sh}(\delta_n k/l) + \mathrm{Bi}\,\mathrm{ch}(\delta_n k/l) - \mathrm{Bi}\,\mathrm{ch}(\delta_n k/l)\,Z}{\delta_n k\,\mathrm{sh}(\delta_n k/l) + \mathrm{Bi}\,\mathrm{ch}(\delta_n k/l)}\ , \tag{10}$$

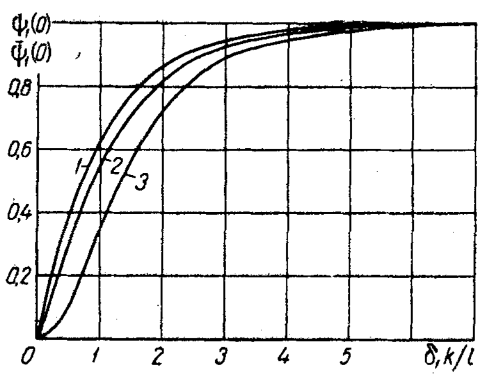
δ_n are roots of the characteristic equation

$$\frac{W_0(\delta_n k)}{W_1(\delta_n k)} = \frac{\delta_n k}{\mathrm{Bi}}\ ,$$
$$W_0(\delta_n R) = I_0(\delta_n R)\,Y_1(\delta_n) - I_1(\delta_n)\,Y_0(\delta_n R). \tag{11}$$

When $\psi_n(Z) \to 1$, the solution for the plane Z = 0 ceases to depend on cylinder height and transforms into the solution of the corresponding one-dimensional problem, which consequently has the form

$$T(R,\ Z,\ \mathrm{Fo}) = 1 + \mathrm{Pd\,Fo} - 2\sum_{n=1}^{\infty} \varphi_n(R). \tag{12}$$

It is thus obvious that to choose the specimen dimensions, it is necessary to analyze the function $\psi_n(0)$, having determined the values of the complex $\delta_n k/l$ for which the value of this function is fairly close to unity. Then, assigning definite values of R_2 and k, we can determine h as well.



Graph of the relations $\psi_1(0) = f\left(\frac{\delta_1 k}{e}\right)$ and $\bar{\psi}_1(0) = f\left(\frac{\delta_1 k}{l}\right)$ with 1) Bi = 1.5, 2) Bi = 3.0, 3) Bi $\to \infty$.

The figure shows the relation $\psi_1(0) = f\left(\frac{\delta_1 k}{l}\right)$.

Curve 1 corresponds to Bi = 1.5, and curve 2 to Bi = 3.0. It can be seen from the graph that when $\delta_1 k/l = 5.5$ for the case Bi = 1.5 and $\delta_1 k/l = 5.7$ for the case Bi = 3.0, the function $\psi_1(0)$ becomes equal to one with an accuracy, as calculations show, of 0.5%. Since experiments to determine the thermophysical properties are carried out in the given case with specimens for which k = 20, while the outside radius R_2 equals 15×10^{-3} m it is easy to calculate that the half-height h for such specimens is 0.056 m when Bi = 1.5 and 0.046 m when Bi = 3.0.

These calculations were carried out using values of the first roots in expression for $\psi_1(0)$ ($\delta_1 = 0.073$ when Bi = 1.5 and $\delta_1 = 0.090$ when Bi = 3.0). All subsequent roots of the characteristic equation (11) for the above two values of Bi bring the value of $\psi_n(0)$ close to unity for even smaller values of $\delta_n k/l$.

It is interesting to calculate theoretically the influence of axial heat fluxes on the temperature field of the cylinder when Bi $\to \infty$. This is related with the fact that in investigations of thermophysical properties at high temperatures, transfer of heat to the test specimen results mainly from radiation, which finally leads to the degeneration of the boundary condition of the third kind at the outer surface into one of the first kind. Experimentally, this means that it is necessary to maintain a linear increase in the temperature not of the medium, but of the outer surface of the test specimen.

The solution of the corresponding problem is analogous to that given above, but with boundary conditions (4) and (6) replaced by the conditions

$$T|_{R=k} = 1 + \mathrm{Pd\,Fo}\,, \tag{13}$$

$$T|_{Z=1} = 1 + \mathrm{Pd\,Fo}, \tag{14}$$

and has the form

$$T(R,\ Z,\ \mathrm{Fo}) = 1 + \mathrm{Pd}\,\mathrm{Fo} - F(R,\ Z), \tag{15}$$

where

$$F(R,\ Z) = 2\pi^2 \sum_{n=1}^{\infty} \bar{\varphi}_n(R)\,\bar{\psi}_n(Z), \tag{16}$$

$$\bar{\varphi}_n(R) = \left[\frac{\mathrm{Pd}\,W_1(\delta_n k)}{\delta_n k} + \frac{2\mathrm{Ki}}{\delta_n \pi}\right] \frac{W_0(\delta_n R)}{\pi^2 k^2 \delta_n^2 W_1^2(\delta_n k) - 4}, \tag{17}$$

$$\bar{\psi}_n(Z) = 1 - \frac{\mathrm{ch}(\delta_n k/l)\,Z}{\mathrm{ch}(\delta_n k/l)}, \tag{18}$$

and δ_n are roots of the characteristic equation

$$W_0(\delta_n k) = I_0(\delta_n k)\,Y_1(\delta_n) - Y_0(\delta_n k)\,I_1(\delta_n) = 0. \tag{19}$$

When $\bar{\psi}_1(Z) \to 0$ the solution for the plane Z = 0 ceases to depend on the height of the cylinder and transforms into the solution of the corresponding one-dimensional problem, which has the form

$$T(R,\ Z,\ \mathrm{Fo}) = 1 + \mathrm{Pd}\,\mathrm{Fo} - 2\pi^2 \sum_{n=1}^{\infty} \bar{\varphi}_n(R). \tag{20}$$

Obviously, in analyzing this problem, in order to estimate the optimal dimensions of the test specimens, it is necessary to examine $\bar{\psi}_n(Z)$ at Z = 0.

Curve 3 in the figure represents the relation $\bar{\psi}_1(0) = f\left(\frac{\delta_n k}{l}\right)$. It can be seen from the graph that when $\delta_1 k/l$ = = 6.4, $\bar{\psi}_1(0)$ becomes equal to unity with an accuracy of not less than 0.5%. For specimens with the above values of R_2 and k, the calculated value of the cylinder half-length h is 0.040 m.

This calculation used a value δ_1 = 0.121 for the first root of characteristic equation (19).

Thus, using cylindrical specimens with a ratio h/R_2 = 3.0-4.0, one can, with sufficient accuracy, calculate the thermophysical properties from the formulas obtained by solving the one-dimensional problem formulated in [5].

NOTATION

t – temperature; t_0 – initial temperature; r – radius; R_1 – inside radius of cylinder; z – height; h – half-length of cylinder; R_2 – outside radius of cylinder; τ – time; a – thermal diffusivity; λ – thermal conductivity; q – specific heat flux; b – rate of heating; α – heat transfer coefficient; $T = t/t_0$; $R = r/R_1$; $Z = z/h$; $l = R_2/h$; $k = R_2/R_1$; $\mathrm{Fo} = a\tau/R_2^2$ – Fourier number; $\mathrm{Bi} = \alpha R_2/\lambda$ – Biot number; $\mathrm{Pd} = bR_2^2/at_0$ – Predvoditelev number; $\mathrm{Ki} = qR_1/\lambda t_0$ – Kirpichev number.

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18 June 1964

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